On the Evaluation of Reachable Workspace for Redundant Manipulators

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Abstract

In this paper, we discuss the problem of computing the reachable workspace for redundant manipulators. Algorithms that compute workspace boundary points by using screw theory are presented. These algorithms cannot distinguish holes and voids that are buried within the reachable workspace. We present an algorithm that utilizes inverse kinematics in order to detect the unreachable regions (holes and voids) within the reachable workspace.

1 Introduction

The reachable workspace of a manipulator is the volume or space encompassing all points that a reference point P on the end effector traces as all the joints move through their respective ranges of motion [4, 9]. The problem of computing the workspace for a redundant manipulator has applications in a variety of fields such as robotics, computer aided design, and computer graphics. Although the workspace problem has long been on the agenda of researchers in robotics, they have not formulated a satisfactory and general solution. A workspace is said to have a hole if there exist at least one straight line which is surrounded by the workspace yet without making contact with it [14, 6]. A workspace is said to have a void if there exist a closed region R, buried within the reachable workspace, such that all points inside the bounding surface of R are not reached by the manipulator [14, 6].

In the next two sections, we describe algorithms that use screw theory and inverse kinematics in order to compute the reachable workspace. Each class of these algorithms has advantages and disadvantages. Instead of debating the merits of these algorithms, we integrate them as described in section four.

2 Algorithms based on screw theory

Kumar [4, 5] did pioneering work in computing workspace boundary points by using screw theory. He used the fact that the manipulator is in singular configuration when the end effector reference point is positioned at a workspace boundary point. This configuration occurs when all active[1] joint axes are all reciprocal to a zero pitch wrench (force)

[1] A joint is termed inactive when it reaches one of its limits.

axis[2] [5, 12, 13, 11]. This is evident because the wrench will create moments about the joint axes that are not reciprocal to the wrench axis if the reciprocal condition is not satisfied. Accordingly, these moments cause those joints to move until the reciprocal condition is satisfied or until they reach one of their limits. For a revolute joint (screw axis of zero pitch), the reciprocal condition is satisfied when the wrench axis has either finite or infinite intersection with the joint axis [5, 11]. For a prismatic joint, the reciprocal condition is met when the wrench axis is perpendicular to the joint axis [11].

A wrench of zero pitch (force) is applied to a reference point on the end effector in order to compute a workspace boundary point in the force direction. Then, the direction of the force is changed to sweep either the entire workspace boundary. A closed form algorithm that computes the joint variables satisfying the above conditions is used. The algorithm generates 2^{n-1} different surfaces which bound different workspaces for a manipulator of n joints. These different surfaces result from the fact that each joint can assume one of two positions under the force application. One of these positions corresponds to a stable equilibrium while the other corresponds to unstable equilibrium. These two positions can be distinguished by computing the work done by the applied force to the end effector when the joint is disturbed from its equilibrium position. If the work done is positive, the disturbed joint is in unstable equilibrium since the force continues doing work until the joint takes up a stable equilibrium position. On the other hand, if the work done is negative, the disturbed joint is in equilibrium position since the applied force causes the joint to return to its initial position when the disturbing torque is removed. Hence, this type of algorithms traces re-entrant surfaces as well as the boundary surfaces. These reentrant surfaces are non-crossable when the manipulator is positioned in the configuration that traces those surfaces, i.e., they represent barriers inside the workspace and affect the manipulator's controllability. The following example illustrates this approach.

Example

Consider a planar manipulator that has three ideal revolute joints[3] with parallel axes as shown in figure 1. A force is applied to a reference point in the end effector. The manipulator will be in its extended positions when the force line of action intersects all joints' axes. The contours which bound envelope of the workspace as well as the interior surfaces result from rotating these extended positions about the first joint axis. Figure 2 illustrates the four different workspaces. The workspace envelope results when all joints are in equilibrium positions.

[2]In screw theory, two screw axes are called reciprocal to each other when the wrench applied about one screw axis does no work about the other screw axis.

[3]We have used a three degree of freedom planar manipulator since it is simple to illustrate and its behavior is similar to a six degrees of freedom manipulator in space.

Finally, this class of algorithms provide surface classification based on the stability of different joints under the force application; however, it cannot detect holes and voids in the workspace. The ability to distinguish holes and voids that are buried inside the workspace can be achieved by using inverse kinematics as described in section four.

3 Algorithms based on inverse kinematics

The inverse kinematics problem involves computing the set of joint variables that would place the end effector in a prespecified position and orientation. The inverse kinematics is not as simple as the forward kinematics. Because the kinematic equations are nonlinear, their solutions are not always easy or even possible in closed form. Also, the questions of existence of a solution, and of multiple solutions arise. The existence or nonexistence of a kinematic solution defines the workspace of a given articulated chain. The lack of a solution means that the chain can not attain the desired position and orientation because it lies outside the workspace. The inverse kinematic problem (IKP) can be solved directly only if the considered chain is kinematically simple [7]. If the number of links is greater than six, the only available approach to date is to use a numerical method to approximate the actual solution. The inverse kinematics problem is modeled as a nonlinear programming problem. The objective of this problem is to minimize a non-negative potential function P. This function represents the difference between the current position and orientation of the end effector and the goal[4]. This function has the value zero if the goal is achieved. If the goal lies outside the workspace, the procedure will return the joint angle values that will position the end effector at the closest point to the goal which lies on the reachable workspace envelope. The potential function P is a function of the position and orientation of the end effector.

$$P = P(r(Q), v_1(Q), v_2(Q))$$

where:
r: is the position vector.
v_1, v_2: are two unit vectors that determine the end effector orientation.
Q: is the vector of the joint angles.

The joint limits q_{imin}, q_{imax}, the lower and the upper limits on q_i respectively, are described by linear constraints.

Based on the above discussion, the inverse kinematics problem is formulated as follows:

$$\text{Min } P(Q) \qquad (1)$$

subject to:

$$q_{imin} \leq q_i \leq q_{imax} \quad \text{for} \quad 1 \leq i \leq n \qquad (2)$$

The algorithm used to solve this problem is described in detail in [15]. It uses Davidon's

[4]A goal is the desired position and orientation of the end effector.

variable metric method with BFGS (Broyden, Fletcher, Goldfarb, Shanno) [1, 3, 10] approximate Hessian matrix update formula and Rosen's projection method to handle the linear constraints [3, 2]. The method is superlinear convergent [8] and each iteration has a complexity of $O(n^2)$ where n is the total number of joint angles in the articulated chain.

The major problems with this class of algorithms are:

1. The optimal solution is not guaranteed since it may converge to a local minimum rather than the global minimum.
2. The iteration time is significantly affected by the value of the tolerance.

4 Hybrid Algorithms

We have discussed how algorithms based on screw theory are used to compute different workspace contours. However, this class of algorithms does not offer a criteria to distinguish holes and voids in the generated workspace, i.e., there is no way to examine whether the points bounded by different contours in figure 2 are reachable or not. The ability to distinguish holes and voids that are buried inside the workspace can be achieved by using the inverse kinematics based algorithms for points that lie between different interior surfaces. An interior surface is a hole (or void) if it lies between a reachable point and a nonreachable one. The following example illustrates this approach.

Example

Figure 2 illustrates a three degrees of freedom planar manipulator. We can decide whether the different contours are workspace boundaries by using an inverse kinematics based algorithm. This is achieved by calling the inverse kinematics algorithm in order to test the reachability of points A, B, C, and D. The inner most contour belongs to an interior workspace boundary since point B is reachable and A is not. The other contours are interior since points B, C and D are reachable.

5 Conclusions

Algorithms based on screw theory are used to compute workspace contours. These contours include the workspace boundary and interior surfaces. These contours are classified by the stability and unstability of the manipulator's joints. However, this classification does not provide any information about holes and voids inside the workspace. We illustrated how to integrate those algorithms and the inverse kinematics into a "hybrid algorithm". The hybrid algorithms can detect the holes and voids that are buried inside the workspace.

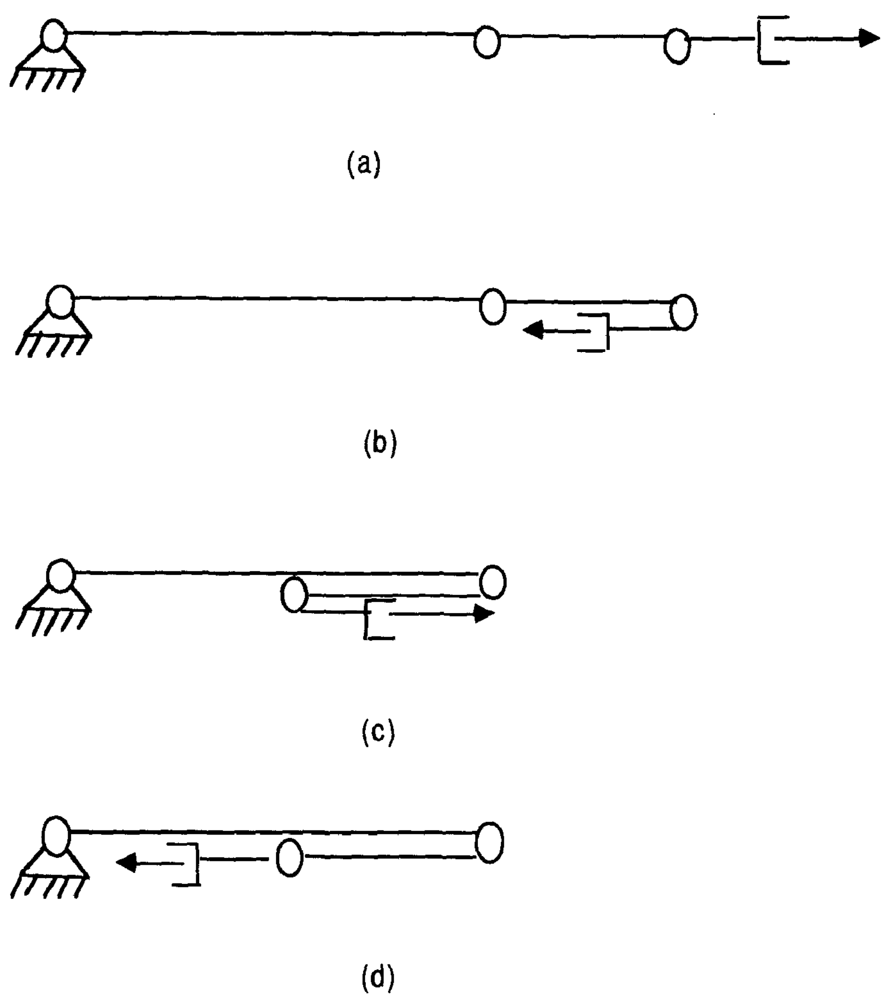

Figure 1: Stable and unstable equilibrium positions

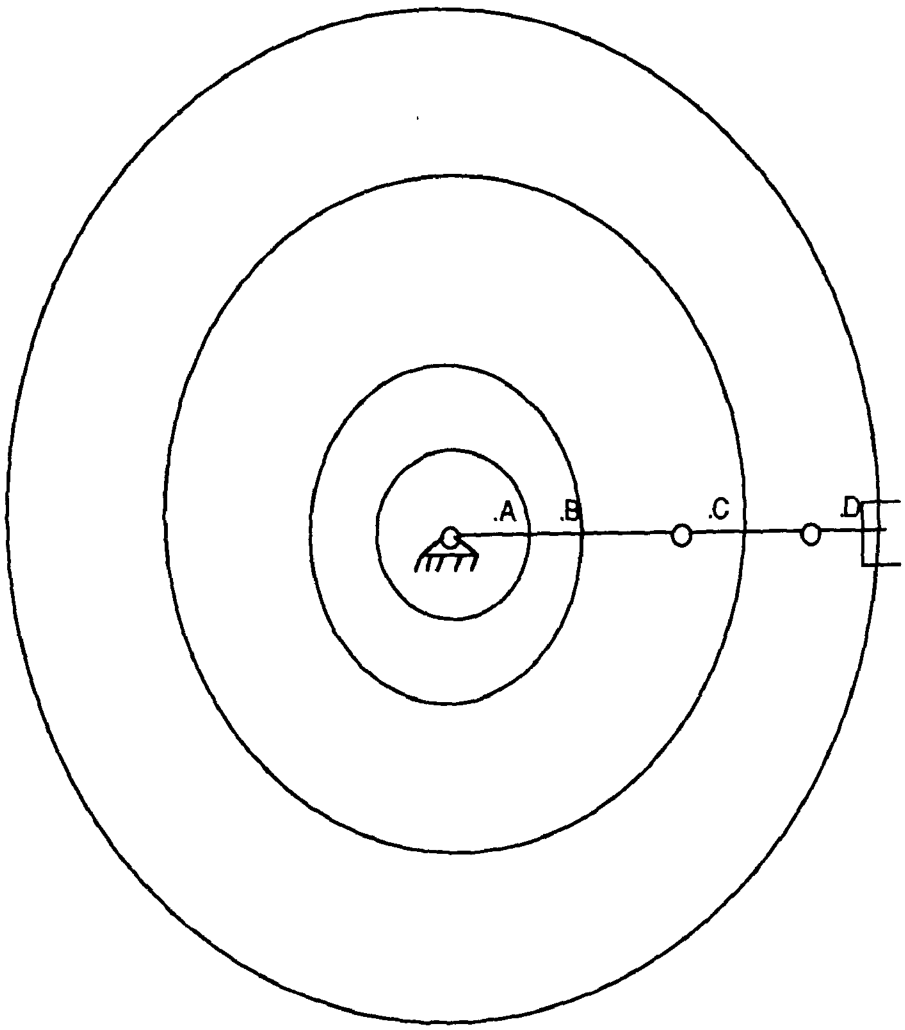

Figure 2: Different workspace contours

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